

# NEW INSIGHT INTO THE FLUCTUATIONS OF THE MOVING VORTEX LATTICE: NON-GAUSSIAN NOISE AND LÉVY FLIGHTS

J. Scola\*, A. Pautrat, C. Goupil, Ch. Simon

*CRISMAT/ENSICAEN et Université de Caen, CNRS UMR 6508, 14050 Caen, France,*

B. Domengès

*LAMIP, Phillips/ENSICAEN, 14050 Caen, France*

C. Villard

*CRTBT/CRETA, CNRS UPR 5001, 38042 Grenoble, France*

We report measurements and analysis of the voltage noise due to vortex motion, performed in superconducting Niobium micro-bridges. Noise in such small systems exhibits important changes from the behavior commonly reported in macroscopic samples. In the low biasing current regime, the voltage fluctuations are shown to deviate substantially from the Gaussian behavior which is systematically observed at higher currents in the so called flux-flow regime. The responsibility of the spatial inhomogeneities of the critical current in this deviation from Gaussian behavior is emphasized. We also report on the first investigation of the effect of an artificial pinning array on the voltage noise statistics. It is shown that the fluctuations can lose their stationarity, and exhibit a Lévy flight-like behavior.

*Keywords:* Vortex dynamics; non-Gaussian noise; Lévy flight.

## 1. On vortex noise

Dynamic properties of a current-driven vortex lattice are determined by the competition between two antagonist mechanisms: on the one hand, the random pinning potential, responsible for the critical current, tends to distort the vortex lattice. But on the other hand, the long-range magnetic interactions between vortices are responsible for their collective behavior and tend to stabilize a lattice. The way these two interactions balance each other is still under debate, and is the subject of numerous theoretical predictions, not only for vortex lattices, but also in the larger field of disordered elastic systems [1]. Experimentally, the observation of Bragg peaks gives evidence for the persistence of a long range order during vortex motion (the flux-flow regime) [2, 3]. However, this moving lattice can not be completely perfect because it keeps on interacting with its pinning potential, as evidenced by the persistence of a main critical current  $I_c$ , even at high biasing current (e.g. [4, 5]).

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\*Present address: Service de Physique de l'État Condensé, CNRS URA 2464, DSM/DRECAM/SPEC, CEA Saclay, 91191 Gif sur Yvette Cedex, France. joseph.scola@cea.fr

These dynamic interactions arise in very small fluctuations of the lattice velocity ( $\delta\mathbf{v}_L$ ) and of the magnetic field density ( $\propto \delta\mathbf{B}$ ). The measurement and analysis of these fluctuations can give important information on the underlying static and dynamic features.

Experimentally, vortex dynamics can be easily probed by transport measurements: the flow of the current-driven vortex lattice generates an electric field in the sample ( $\mathbf{E} = -\mathbf{v}_L \times \mathbf{B}$ ). The fluctuations induced by the pinning interactions are reflected into the measurable voltage drop across the sample:

$$\delta V(t) = - \int (\delta\mathbf{v}_L(t) \times \overline{\mathbf{B}} + \delta\mathbf{B}(t) \times \overline{\mathbf{v}_L}) \cdot d\mathbf{l},$$

where the overlined symbols stand for the mean values. Since the  $\delta V(t)$  signal results from a random mechanism, it is processed as a noise. Most of the studies carried out on vortex noise have focused on the power spectral density (PSD), which gives partial information on the phenomenon. In order to improve the description of the noise process, the analysis of its power distribution is sometimes necessary. It is generally assumed to be Gaussian [6], as it is expected for independent fluctuators due to the general applicability of the Central Limit Theorem (CLT). It is interesting to notice the striking exception that takes place in the low- $T_c$  superconductor  $2H\text{-NbSe}_2$  in the vicinity of the peak effect (i.e. an anomalous increase of  $I_c$  at high field). Two macroscopic vortex states with different pinning properties have been observed to coexist in this regime [7, 3]. At the same time, the noise is unconventional, marked by an extremely high level, and exhibits strong non-Gaussian features [8, 9]. These combined effects have been explained using a model of random creation and annihilation of metastable vortex states [10]. To our knowledge, this peculiar case, where macroscopic inhomogeneities are present, is the sole direct measurement of a clear deviation from Gaussian fluctuations in the moving vortex state. Since vortex noise is most generally recorded in large systems (typically centimetric), statistical averaging can make the deviation very difficult to observe. Here, we study small bridges where fluctuations are expected to be fewer. In such small systems, non-Gaussian effects are more likely observable [11].

The experiments were performed on micro-bridges made on a 450nm-thick film of Nb, which was ion-beam deposited on a sapphire substrate. The pinning properties and superconducting parameters of similar samples are given in ref. [12]. The films present a relatively smooth surface (rms roughness is less than 5nm at the scale of few microns). In order to create an artificial pinning potential for the vortices, twelve rectangular grooves ( $12\mu\text{m} \times 30\text{nm}$ , 30nm deep) regularly spaced by 300nm have been etched at the surface of one bridge, using a focused ion beam. In our experimental configuration, the grooves are parallel to the current direction, i.e. perpendicular to the vortex velocity direction (as sketched in fig. 2). The magnetic field which creates the vortex lattice is applied perpendicular to the film surface. The experimental geometry is summarized in the figure 1.

One of the aims of the experiment was to compare the vortex noise properties between the regular micro-bridge and the micro-bridge with the artificial surface state. For this purpose, we have performed a systematic analysis of the voltage noise, with a particular care on its statistics. The voltage time series were measured with the four probes method. The detailed set-up can be found in the ref. [13]. The raw

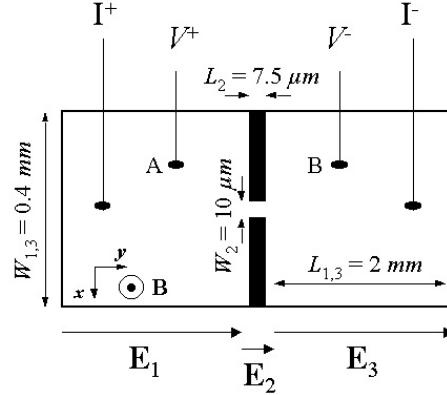


Fig 1. Experimental geometry: the current flows along the  $y$  axis, while the vortices flow along the  $x$  axis. The constriction (heavy black lines) magnifies the current density and lowers the critical current so that the lattice within the constriction depins for smaller current. In our experimental conditions, only the vortices in the constriction (so-called micro-bridge) move, the rest of the lattice being immobile and noiseless. The grooves are etched at the center of the constriction, along the  $y$  axis.

time signal was divided into segments of  $\Delta t = 0.3s$ , each of them being Fourier-transformed afterwards. This gives the time series  $S[f, t]$  of power spectra  $S(f)$ . We have calculated the rms value  $\delta V_{rms}^2 = \int \overline{S}(f) df$ , the mean power spectrum  $\overline{S}(f)$  as a function of the acquisition time, and the fractional histograms (i.e. the distribution of the power contained in fixed frequency windows). We have also calculated the second spectrum:

$$S_2(f, f_2) = \sum_{t=0}^{T_2} S[f, t] e^{-2i\pi f_2 t},$$

with  $f_2$  the second order frequency. If the fluctuations are not correlated on long time-scales (typically longer than 1s with our resolution),  $S_2(f, f_2)$  is expected to be frequency-independent for any  $f$ . Conversely, if they exist, such correlations will lead to a  $f_2$ -dependency in the second spectrum. We use this tool to estimate if any long time-scale correlations exist in the fluctuations. It offers more robustness than a direct correlation calculation for rather low signal-to-noise ratios, which is our case here.

## 2. Results and discussion

To begin with, we briefly describe the noise for  $I \gg I_c$ , in the flux-flow regime. The flux-flow regime is characterized by the linear relation  $V = R_f(I - I_c)$ ,  $R_f$  being the flux-flow resistance of the vortex lattice. Since the flux-flow noise properties have been observed to be similar for all micro-bridges, regardless of their surface state and edges state, the samples will not be distinguished for the moment. In general, the introduction of a periodic artificial pinning potential leads to the observation of matching fields, when the magnetic field matches exactly an integer number of vortices between two pinning sites [14]. We have observed a substantial increase

of the critical current at matching fields in the grooved sample [15]. But, in order to avoid any noise artifacts, the low dissipation requirement imposes to perform measurements only at fields much higher than the matching fields, where the critical current is moderate. These matching effects can thus be ignored here.

The flux-flow noise can be described as follows: the rms value hardly depends on the driving current, and hence on the mean lattice velocity  $v_L = E/B = \rho_f(J - J_c)/B$ , (fig. 2 and 3), and it varies like the product  $R_f I_c$  (inset of fig. 2); the noise spectrum ranges roughly below 1kHz; fractional histograms look clearly Gaussian (fig. 4). All those properties also depict the flux-flow noise in bulk samples [4,13]. These results are of importance, given that in a micro-bridge the vortex velocity  $v_L$  can be much higher than in bulk samples, due to the largest values of the driving current density ( $J - J_c$ ). Here,  $v_L$  is typically three orders of magnitude higher than in bulk samples (up to  $85m/s$  for  $I = 10A$  compared to  $v_L \lesssim 0.1m/s$  in bulk Nb). This clearly shows that the vortex lattice velocity plays a minor role in the low frequency noise mechanism. This contradicts the common descriptions of the noise coming from direct interactions of the vortices with some bulk pinning centers. In such approaches, the noise should disappear at high speed, as the pinning interactions are expected to vanish (e.g. [16,17]). In contrast, the observed properties support a description of the noise in terms of surface critical current fluctuations, which has been verified in bulk samples of low- $T_c$  superconductors [4,18].

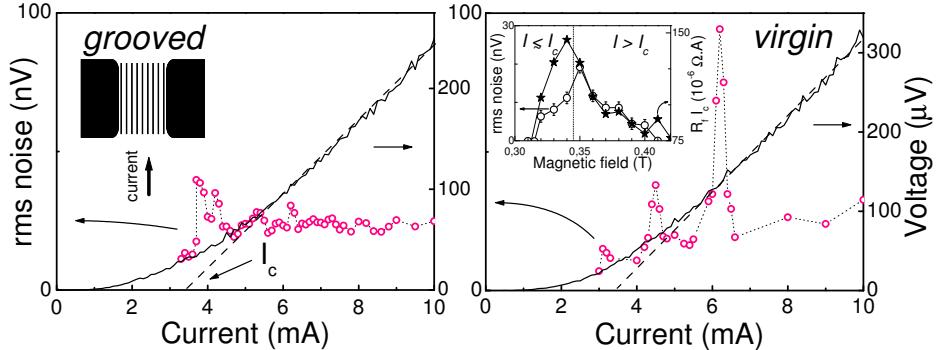


Fig 2. Average voltage (solid line) along with the rms noise power  $\delta V_{rms}$  (○) plotted against the driving current  $I$ , for the grooved and virgin bridges (the depinning noise points in the former are shown as an indication). The linear extrapolation (dashed line) of the flux-flow voltage intercepts the  $I$ -axis at the overall critical current  $I_c$ . The etched pattern on the micro-bridge is sketched on the left-hand graph. Insert: magnetic field scan of the rms noise (○) at  $I = 7mA$ , and the product  $R_f I_c$  (★). On the right-hand side of the vertical line, the lattice is in flux-flow regime.  $T = 4.2K$ ,  $B = 0.35T$ .

We will now focus on the low current regime. This corresponds to the nonlinear part of the  $V(I)$  curve, where it is known that the vortex depinning is not homogeneous. We observe that the rms noise exhibits peaks for some current values (fig. 2 and 3) in most micro-bridges. This excess noise can be associated to inhomogeneities of the flow before the flux-flow is reached. Such inhomogeneities are likely to come from a non-uniform critical current. To elucidate the origin of

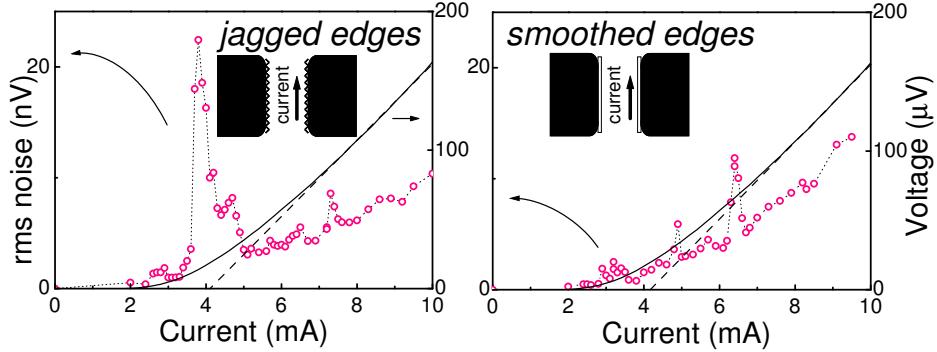


Fig 3. Average voltage and rms noise versus the driving current, as in figure 2. The types of edges are sketched. The etched pattern is made of two series of forty  $200\text{nm} \times 200\text{nm}$  squares (technical details can be found in ref. [15]).  $B = 0.32\text{T}$ .

the peaks, micro-bridges were modified using a focused ion beam. The edges of one micro-bridge were perfectly smoothed at the scale of the lattice, whereas the edges of another one were periodically jagged (see schematic drawing in figure 3). None of the critical current, the flux-flow resistivity, and the flux-flow noise behavior (weak monotonic dependency on  $I$ ) were affected by the changes of the edges shape in our experimental conditions. However, at low currents, the noise peaks are almost absent for the smoothed edges whereas a huge peak is visible for the jagged edges (figure 3). These results show that the depinning peaks are likely due to the irregularities of the edges. Note that the Oersted field created by the transport current does vary along the jagged edges, but it is too small to explain our results: using the Biot-Savart formula, one estimates that the magnetic field at the edges induced by  $10\text{mA}$  in our micro-bridges is only  $10^{-3}\text{T}$ , which is hundred times smaller than the external magnetic field. Our interpretation of the phenomenon is that the edges irregularities cause a modulation of the width, and thereby of the critical current, along the micro-bridge. The critical current is the lowest in the narrowest regions. Conversely, the over-critical current, that is the driving force, is the highest in these locations. Hence, the lattice in the narrowest regions is depinned for the lowest currents. Shear areas are introduced and disappear gradually until the whole lattice flows. It must be noted that the peaks always appear at the same current value, even if the current was increased from zero, or decreased from the flux-flow, or even after an over-critical temperature cycle. This shows that these noise peaks are not history dependent, in agreement with their proposed topographical origin.

To go deeper in the understanding of these noise peaks, we have investigated their statistics. The associated histograms are clearly non-Gaussian (fig. 4-c et d): the noise power is distributed over a much larger range than in the flux-flow regime, while the mean values are comparable. The noise power measured in the grooved micro-bridge is the most unique: it exhibits roughly a power law distribution, within the experimental range (inset of 4-d). To reveal the existence of long time correlations between the fluctuations, the use of the second spectrum is an efficient and complementary tool. The second spectra are observed to be frequency-independent

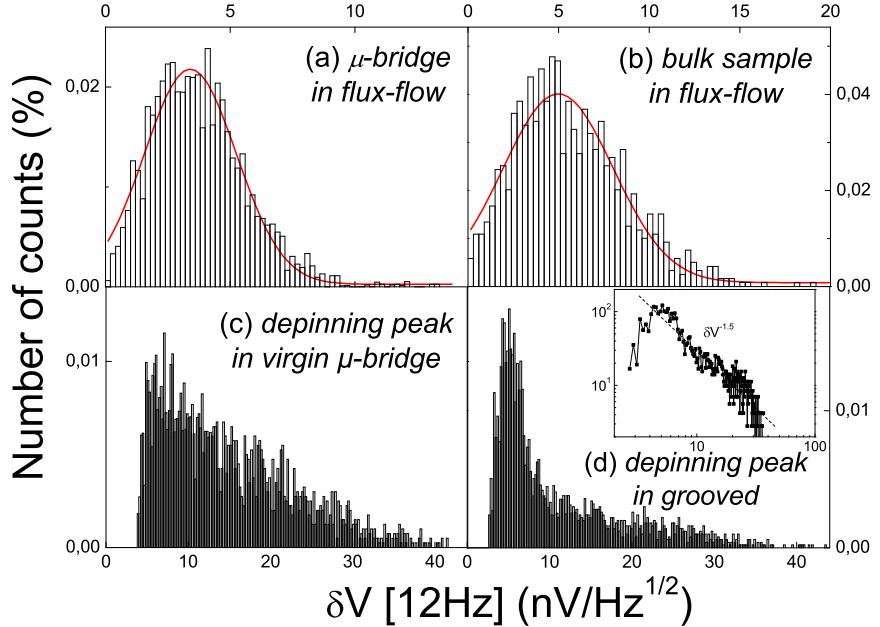


Fig 4. Normalized histograms of the noise magnitude at  $12 \pm 2 \text{ Hz}$  in micro-bridges (a,c,d, 7000 counts) and a bulk Nb sample (b,  $12.10 \times 1.23 \times 0.22 \text{ mm}^3$ , 1200 counts) built from four 30 minute long series, for different driving conditions: (a)  $I = 7.5 \text{ mA}$ ; (b)  $I = 4.4 \text{ A} \gg I_c = 1.61 \text{ A}$ ,  $H = 0.28 \text{ T}$ ; (c)  $I = 4.5 \text{ mA}$ ; (d)  $I = 6 \text{ mA}$ ,  $T = 4.2 \text{ K}$ . Solid curves denote the Gaussian fits. Details on the measuring condition of the bulk sample can be found in [18]. Inset: log-log plot of the histogram (d).

like in flux-flow, despite the presence of extreme values in this regime (fig. 5-a): this reveals the absence of long-time correlations (below  $1 \text{ Hz}$ ). This means that when the applied current sets the lattice in a depinning peak, extreme noise values are reached in an abrupt fashion, rather than a gradual one. In summary, the fluctuations in the depinning peaks can be characterized as erratic and intermittent, without memory. Outside the peaks, the histograms asymmetry is less pronounced, as though an intermediate regime between depinning and flux-flow took place. The two micro-bridges having their edges altered exhibit exactly the same statistic features as the virgin sample (non-Gaussian in the depinning peaks, when they exist, and gaussian in flux-flow).

The existence of non-Gaussian statistics during the depinning shows the difference of vortex flow between flux-flow and depinning. Up to now, the different dynamic regimes of the vortex lattice have been investigated by means of numerical simulations (see e.g. [17]), and very little information have been collected from experiments. Here, we bring an experimental evidence of the distinction between two dynamical regimes. Our results can be interpreted in a similar way as the ex-

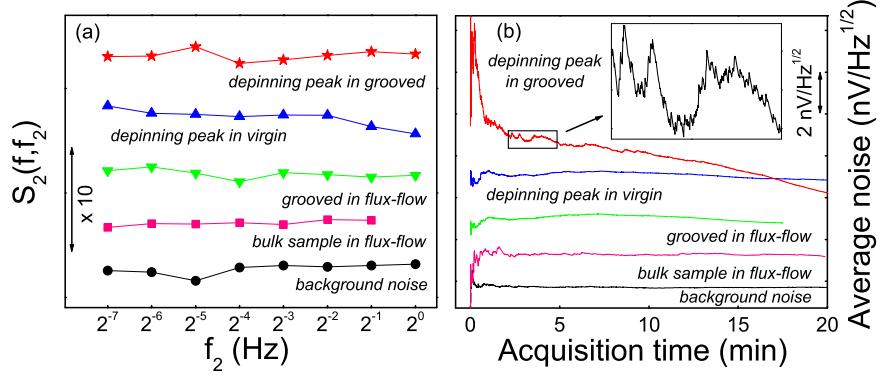


Fig 5. (a): Second spectra of the noise power at 12Hz, represented by octaves in log-log scale. (b): average noise at 12Hz plotted against the acquisition duration; an arbitrary offset is added for presentation purposes. Both quantities are reported for different samples, dynamic regimes and surface states. Calculations at 72Hz yield similar results.

periment carried out on NbSe<sub>2</sub>. Regardless of the discrepancies between Nb micro-bridges and NbSe<sub>2</sub> samples, it seems that here again, the non-Gaussian effects can be associated to spatial inhomogeneities of the critical current [8]. In our case, the fluctuations can not develop over the whole lattice like in flux-flow, because they are confined inside independent lattice slices. The non-Gaussian effects are likely to be enhanced by the smallness of the micro-bridges that limits the number of shear areas. Given the fact that a sum of independent random variables tends to an approximate Gaussian distribution from about five terms, we conclude that, following our interpretation, only one or two shear areas take place simultaneously, and the flux-flow is reached after a few depinning steps. This sets an upper size of a few microns for one slice.

We now focus on the particular effects linked to the presence of the etched grooves. Albeit the depinning noise regimes with or without the grooves look qualitatively similar (non-Gaussian power distributions and white second spectra, as shown in fig. 4 and 5-a), the sample with the grooves exhibits new and interesting features. In order to understand the differences, average noise powers, at a given frequency, are plotted as a function of the acquisition time  $T_2$  under various conditions (fig. 5-b). For stationary noise, no time dependency is expected. This is clearly observed for all samples in flux-flow, and also for the virgin sample in the low current regime. But for the sample with the grooves, this low current noise dramatically drifts with time. This drift occurs at any observable frequency, at any current in the depinning regime, and disappears in flux-flow. It is important to note that this non stationary noise appears only for bridges presenting a long power law tail in the noise power distribution. This probability density decays as  $1/x^\alpha$ , with  $\alpha = 1.5 < 2$ . Such a slow decay magnifies the weight of rare events and the high values. In this case, the statistics is not described by a Gaussian law, but

by a Lévy law, and the central limit theorem (CLT) does not apply anymore [19]. As a consequence of the domination of a small number of terms, the process ergodicity is lost. There is no convergence towards any constant value, as compared to what we had measured in the grooved sample during the depinning (fig. 5-b inset). The dedicated term of such a random walk with a long tail distribution is the Lévy flight. Levy flights can be observed for some extreme cases of anomalous diffusion in an Arrhenius cascade. For such an activated system in a tilted potential landscape, and in the limit where the potential wells are high compared to the activation energy, the trapping time does not converge towards any constant value [20]. It is clear that in our case the diffusion of the fluctuations are not thermally driven, as in a strict analogy with the Arrhenius cascade. The underlying instability process can be here the local hang-and-release of vortices on the surface, which was shown to be a potential candidate for the noise mechanism [4]. Since the drift of the fluctuations are observed only for the sample with the superficial grooves, this shows unambiguously that they are very effective potential barriers for the vortex lattice flow *fluctuations*. Violations of the CLT experimentally observed in physical systems, and related to Lévy flights, have recently received an increasing interest [19]. These observations are often in direct connection with an anomalous diffusion [21,22]. Thus, the return to the flux-flow regime, where the lattice flows homogeneously at the scale of the system, is consistently marked by the return to stationary statistics (fig. 5-b). From a practical point of view, such a Lévy-flight behavior implies also that the size of the system and the number of measurements can play an important role when measuring vortex lattice properties, in some extreme, but experimentally accessible, cases. More generally, by reducing the size of the system, we managed to observe a somewhat individual behavior, as opposed to the averaged one observable in large systems where the CLT is valid. Our results underline how crucial fluctuations analysis can be for studying physical phenomena at small scale.

To conclude, we have reported a detailed voltage noise investigation in superconducting Nb micro-bridges. Clear experimental signatures distinguished the flux-flow regime at high currents and the depinning regime at low currents. The former is always Gaussian and stationary, which is quite comparable to common observations made in bulk samples. In contrast, the latter is characterized by non-Gaussian effects observable in the presence of spatial variations of the critical current. In addition to these non-Gaussian effects, an artificial surface pinning potential behaves like high potential barriers, and induces an abnormal diffusion of the fluctuations. This is, to our knowledge, the first experimental observation of a Lévy flight-like behavior of a vortex lattice. This experiment is an example of the new statistical behavior that can arise from a drastic reduction of the size of the system.

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